Question 1. Derive the formula \( \neg(A \leftrightarrow \neg A) \) in minimal logic. You may construct the derivation by hand, or use the interactive proving tool from the logic labs. [20 marks]

Question 2. (Velleman, How To Prove It, p. 63) Define suitable signatures for the following statements and formalise them.

(a) If anyone in the dorm has the measles, then everyone who has a friend in the dorm will have to be quarantined.

(b) If anyone can do it, Jones can.

(c) If Jones can do it, anyone can.

[20 marks]

Question 3. (Velleman, How To Prove It, p. 63) Define a suitable signature of the following statements and formalise them. What are the free variables in each statement?

(a) Every number that is larger than \( x \) is larger than \( y \).

(b) For every number \( a \), the equation \( ax^2 + 4x - 2 = 0 \) has exactly one solution iff \( a \geq -2 \).

[20 marks]

Question 4. Consider a system of processes with a binary reduction relation \( R \). If \( R(x, y) \) holds, we say that the process \( x \) reduces to the process \( y \).

A process is said to be in normal form if it cannot be reduced. The reduction relation \( R \) is called weakly normalising if every process that is not in normal form can be reduced to a process in normal form.

A process is called a sink if every process can be reduced to it.

(a) Formalise the statement that \( R \) is weakly normalising.

(b) Formalise the statement that a weakly normalising reduction relation \( R \) cannot have a sink.

[20 marks]

Question 5. Derive the formula you found in Question 4 (b) in minimal predicate logic.

[20 marks]