CS.376 Programming with Abstract Data Types

Additional Exercises (not assessed)

Question 1. Give a natural deduction proof of $\neg \neg \forall x \, P(x) \rightarrow \forall x \, \neg \neg P(x)$ using the rules of
minimal logic only and write down the corresponding proof term.

Question 2. To which proof does the following proof term correspond?

$$
\lambda u_1: P \rightarrow Q \lambda u_2: Q \rightarrow R \lambda u_3: P(u_2(u_1))
$$

Question 3. Consider the signature $\Sigma$ containing the sorts nat and list, the constants $0: \text{nat}$ and
nil: list, and the operations succ: nat $\rightarrow$ nat and cons: nat $\times$ list $\rightarrow$ list, append: list $\times$ list $\rightarrow$ list.
Let $A$ be the $\Sigma$-algebra defined by

$$
A_{\text{nat}} := \mathbb{N} = \text{the set of natural numbers}
$$

$$
A_{\text{list}} := \text{the set of finite lists of natural numbers}
$$

$$
0^A := 0
$$

$$
\text{nil}^A := \text{the empty list}
$$

$$
\text{succ}^A(n) := n + 1 \ (n \in \mathbb{N})
$$

$$
\text{append}^A(l_1, l_2) := \text{the concatenation of } l_1 \text{ and } l_2
$$

(a) Show that $A$ is not initial in the class of all $\Sigma$-algebras.

(b) Find a set $E$ of equations such that $A$ is a model of the initial specification Init–Spec($\Sigma, E$).

Question 4. Let $s$ be a sort, $c: s$ a constant, $f, g: s \times s \rightarrow s$ and $h: s \rightarrow s$ operations, and $x: s$
a variable.

Decide for each of the following term rewriting systems $R_1$, $R_2$ and $R_3$ whether it is terminating:

$$
R_1 := \{ f(x, h(c)) \rightarrow g(x, x) \}
$$

$$
R_2 := \{ g(x, h(c)) \rightarrow f(x, x) \}
$$

$$
R_3 := R_1 \cup R_2
$$

Justify your answers!

Question 5. Let $\text{nat}$ be a sort, $1: \text{nat}$ a constant, $e, *: \text{nat} \times \text{nat} \rightarrow \text{nat}$ operations, and $x, y, z: \text{nat}$
variables. We will use infix notation for $e$. Consider the term rewriting system given by the following rules:

$$
x * 1 \rightarrow x
$$

$$
e(x, 1) \rightarrow x
$$

$$
e(x, y * z) \rightarrow e(e(x, y), z)
$$

Show that this term rewriting system is terminating and confluent.