Solutions to the exercises for Chapter 3 of the Lecture Notes

1. We have to show

   \[ A \models \forall x (\text{first}(\text{cons}(x, l)) = x), \]

   \[ A \models \forall x (\text{rest}(\text{cons}(x, l)) = l) \]

   i.e.

   \[ \text{first}^A(\text{cons}^A(b, l)) = b, \]

   \[ \text{rest}^A(\text{cons}^A(b, l)) = l \]

   for all boolean values \( b \) and all finite lists \( l \) of boolean values. But this is obvious.

2. \( A \) is not initial in the class of all models of \( LIST(BOOLE) \). In order to show this it suffices find an equation between closed terms that holds in \( A \), but does not hold in all models of \( A \). For example the equation

   \[ \text{first}(\text{nil}) = F \]

   holds in \( A \), but it is easy to modify the algebra \( A \) to an algebra \( B \) which is still a model of \( LIST(BOOLE) \), but where the equation above is false (just set \( \text{first}^B([], ) := \#t \)).