Question 1. The algebras $A$ and $B$ are not isomorphic since they do not satisfy the same $\Sigma$-formulas. For example, $\forall x \exists y f(x, y) = c$ holds in $A$, but not in $B$.

The algebras $A$ and $C$ are isomorphic. Here is an isomorphism:

$$\varphi : \mathbb{R} \rightarrow \mathbb{R}^+, \quad \varphi(x) := e^x$$

We know from school mathematics:

1. $\varphi(x) > 0$ for all $x \in \mathbb{R}$ (hence $\varphi : \mathbb{R} \rightarrow \mathbb{R}^+$ is well-defined) and $\varphi$ is bijective (the natural logarithm is its inverse).
2. $\varphi(0) = 1$ and $\varphi(x + y) = \varphi(x) \ast \varphi(y)$, hence $\varphi$ is a homomorphism from $A$ to $C$.

Question 2. (a)
(b) A minimal set of generators is \{0, succ, T, F, singleton, union\}. The generators are not free, since different generator terms may denote the same set. For example, the generator terms \(\text{union}(\text{singleton}(0), \text{singleton}(0))\) and \(\text{singleton}(0)\) are different, but denote the same set \(\{0\}\).

(c) From the equations for \(\text{count}\) and the equations in (a) it follows (assuming suitable equations for addition)

\[
\text{succ}(0) = \text{count}(\{0\}) = \text{count}(\{0\} \cup \{0\}) = \text{count}(\{0\}) + \text{count}(\{0\}) = \text{succ}(0) + \text{succ}(0) = \text{succ}(\text{succ}(0))
\]

which is a new equation for natural numbers. In fact, one could prove that all natural numbers are equal (similarly for booleans and sets). Hence the model of the extended initial specification collapses.

**Question 3.** The given term rewriting system:

\[
\begin{align*}
x \ast 1 & \mapsto x \\
1 \ast x & \mapsto x \\
\exp(1, x) & \mapsto 1 \\
\exp(x, 1) & \mapsto x \\
\exp(x \ast y, z) & \mapsto \exp(x, z) \ast \exp(y, z) \\
\exp(x, y \ast z) & \mapsto \exp(\exp(x, y), z)
\end{align*}
\]

To prove termination we use an interpretation in the following algebra \(A\):

\[
A_s := \{2, 3, 4, \ldots\}, \quad 1_A := 2, \quad n \ast_A m := n \ast m + 1, \quad \exp_A(n, m) := n^m.
\]

Clearly, the operations are monotone. It remains to be checked that \(l_A^\alpha \succ r_A^\alpha\) for every rule \(l \mapsto r\) and every assignment \(\alpha: \text{Variables} \rightarrow A\). Hence, one has to verify for all \(m, n, k \in \{2, 3, 4, \ldots\}\) the following inequalities:

\[
\begin{align*}
n \ast_A 1_A & > n \\
1_A \ast_A n & > n \\
\exp_A(1_A, n) & > 1_A \\
\exp_A(n, 1_A) & > n \\
\exp_A(n \ast_A m, k) & > \exp_A(n, k) \ast_A \exp_A(m, k) \\
\exp_A(n, m \ast_A k) & > \exp_A(\exp_A(n, m), k)
\end{align*}
\]

that is

\[
\begin{align*}
n \ast 2 + 1 & > n \\
2 \ast n + 1 & > n \\
2^n & > 2 \\
n^2 & > n \\
(n \ast m + 1)^k & > n^k \ast m^k + 1 \\
n^{m^k + 1} & > (n^m)^k
\end{align*}
\]

Clearly, all inequalities hold.