CS_376 Programming with Abstract Data Types

Coursework 1

Date due: Tuesday, November 3, 2008

Question 1. Use a boolean operation $\text{att}(s, l, c)$, with the intended meaning “student $s$ attended lecture $l$ of course $c$”, to formally express the following statements:

(a) $\text{reg}(s, c) := \text{student } s \text{ attended course } c \text{ regularly, that is, missed at most one lecture of course } c.$

(b) $\text{model}(s) := \text{student } s \text{ missed at most two lectures in total.}$

(c) $\text{lazy}(s) := \text{student } s \text{ attended no course regularly.}$

(d) No student attended all courses regularly.

(e) Each course which was attended by at least one student regularly had at least one lecture with full attendance.

Write down a possible signature $\Sigma$ for your formulas.

[30 marks]

Question 2. For each of the following $\Sigma$-formulas decide whether it held in the algebra of our Level 2 Computer Science in 2008/9, or whether it is logically valid.

(a) $\forall s \ (\text{model}(s) \rightarrow \neg \text{lazy}(s))$

(b) $\forall s \ (\text{reg}(s, \text{cs221}) \rightarrow \neg \text{lazy}(s))$

If you claim that a formula is not logically valid, give a counterexample, that is, a $\Sigma$-algebra where the formula does not hold.

If you claim that a formula is logically valid, give a natural deduction proof.

[30 marks]
Question 3. Give natural deduction proofs of the following formulas.

(a) $P \land Q \rightarrow P \lor Q$, in minimal logic;
(b) $\neg(P \leftrightarrow \neg P)$, in minimal logic;
(c) $(P \lor Q) \rightarrow (\neg P \rightarrow Q)$, in intuitionistic logic;
(d) $\neg \forall x P(x) \rightarrow \exists x \neg P(x)$, in classical logic;

[40 marks]

Question 4. (voluntary; not assigned)

Let Stability be the formula $\forall x, y (\neg \neg x = y \rightarrow x = y)$. Prove

$$\text{Stability} \vdash_m \neg \neg P \rightarrow P$$

for all formulas $P$ that contain neither $\lor$ nor $\exists$.

Hint: Use induction on formulas.

Submission

In the lecture on Tuesday, 3rd of November, with a signed Coursework Submission slip attached.