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DynaMoVis: Visualization of dynamic models for urban modeling

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Abstract This additional material includes the development of the retail model.

1 Retail model

The UK county of South Yorkshire is the system of interest. Its retail system and how it evolves is vitally important for our understanding of future planning requirements - in this case where to build new shopping centers or modify capacity. The retail system can be aggregated into 19 discrete retail centers as shown in Figure 1 with 94 population centers covering 1.2M people. Modeling allows us to predict how the system will change over time.

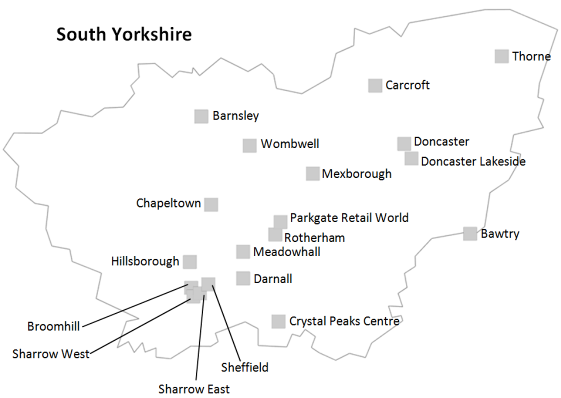

Fig. 1 The South Yorkshire retail system.

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The dynamic urban retail model by [1] is one of the most well known dynamic urban models. This model was one of the first to introduce explicit dynamics and has more recently been presented as the Boltzmann-Lotka-Volterra (BLV) modeling framework [3]. This model provides a framework for exploring nonlinearities, emergence, self-organization and complexity in urban evolution. The model is a high-dimensional nonlinear dynamical system so its behavior is not fully understood. One of the main purposes of visualizing the behavior of this model is to attempt to provide some insights that might help us better understand it and aid further development.

The model works as follows: there are M residential zones, $i = 1, 2, \ldots M$ and N retail zones, $j = 1, 2, \ldots, N$. S_{ij} is the flow of money from residents in zone i to shops in zone j; e_i is the spending per person, and P_i the residential population of zone i. The c_{ij} parameter represents the cost of traveling from residential zone i to retail zone j. The impact of travel cost on consumer flows is modeled by an exponential decay term $e^{-\beta c_{ij}}$. W_j is a measure of the total floorspace of shops in j. The distribution of retail across the system is described by the vector $\mathbf{W} = W_1, \ldots, W_j$. The parameters α and β represent the impact of retail zone size and the impact of travel cost respectively on consumer decisions about where to shop. W_j^α is then the attractiveness of shops in zone j. The amount of money flowing from residents to retail is constrained so that it is equal to the total amount of spending power in the population.

$$\sum_j S_{ij} = e_i P_i \tag{1}$$

Entropy maximising [2] is used to identify the most likely set of consumer flows from each population center

i to each retail center j given the constraints on the system.

$$S_{ij} = A_i e_i P_i W_j^{\alpha} e^{-\beta c_{ij}} \tag{2}$$

where

$$A_i = \frac{1}{\sum_k W_k^{\alpha} e^{-\beta c_{ik}}} \tag{3}$$

This ensures that the constraint in equation 1 is met. The total revenue attracted to retail zone j from all residential zones i is given by:

$$D_j = \sum_i S_{ij} = \sum_i \left[\frac{e_i P_i W_j^{\alpha} e^{-\beta c_{ij}}}{\sum_k W_k^{\alpha} e^{-\beta c_{ik}}} \right] \tag{4}$$

The explicit dynamics in the model are:

$$\frac{dW_j}{dt} = \epsilon \left(D_j - K W_j \right) \tag{5}$$

where K is a constant - the cost per unit of floor space - so that KW_j can be taken as the cost of running the retail zone in j. If the zone is profitable, it grows; if not, it declines. The parameter ϵ determines the speed of response to these signals. These equations may be written as

$$\frac{dW_j}{dt} = \epsilon \left(\sum_i \left[\frac{e_i P_i W_j^{\alpha} e^{-\beta c_{ij}}}{\sum_k W_k^{\alpha} e^{-\beta c_{ik}}} \right] - K W_j \right) \tag{6}$$

Equation 6 is a system of nonlinear simultaneous differential equations, reflecting the nature of relationships between size and revenue in retail systems. These equations cannot be solved analytically: computer simulation and visualization are essential. For simulation purposes, the difference equation form is used:

$$\Delta W_j(t, t+1) = \epsilon \left[D_j(t) - K W_j(t) \right] \tag{7}$$

for the period $(t, t+1)$. Then

$$W_j(t+1) = W_j(t) + \Delta W_j(t, t+1) \tag{8}$$

The equilibrium position is given by

$$D_j = K W_j \tag{9}$$

which also may be written out in full as

$$\sum_i \left[\frac{e_i P_i W_j^{\alpha} e^{-\beta c_{ij}}}{\sum_k W_k^{\alpha} e^{-\beta c_{ik}}} \right] = K W_j \tag{10}$$

Parking charges and other transport factors can have a large impact on the desirability and therefore potential growth of a shopping center and therefore is introduced as m_j which is a multiplier that affects every c_{ij} travel cost into a shopping zone j.

Therefore we arrive that the final equation included in the main paper:

$$\frac{dW_j}{dt} = \epsilon \left(\sum_i \left[\frac{e_i P_i W_j^{\alpha} e^{-\beta m_j c_{ij}}}{\sum_k W_k^{\alpha} e^{-\beta m_j c_{ik}}} \right] - K W_j \right) \tag{11}$$

2 Hyperplane

Looping through $10D$ space at an interval of 21 (arbitrarily chosen) results in $21^{10} = 16.67$ trillion sample locations. Assume each dimension has values 0 to 20 without loss of generality. Each sample will be a $10D$ vector $x_1, x_2, ..., x_{10}$ with each $0 \leq x_i \leq 20$. The sum of the vector is $T = \sum_{i=1}^{10} x_i$. If the vector represents something in the model, e.g., spending power, then T is the total (spending power of the model). During visualization of a complete domain simulation for a $3D$ system, we encountered the situation of Fig. 1, where paths quickly converge to a (hyper)plane. We realized this is because money in the system is assumed to be fixed as part of the simulation equations. Therefore we observed we only needed to seed points close or on the hyperplane to discover the system.

Therefore we need to find all start points where T is equal to the constant amount of money in the system. Given x_1 $(0 \leq x_1 \leq 20)$, the remaining spend is $T - x_1$ which needs to be distributed over dimensions $x_2, ...x_{10}$. Therefore x_2 can have values 0 to the minimum of 20 and $T - x_1$. The remaining total to distribute over $x_3, ..., x_{10}$ is $T - x_1 - x_2$. This continues until T is distributed across all dimensions. Code to implement this (along with a full enumeration) is included as C++ in the additional material.

References

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